COORDINATED REPLENISHMENT POLICIES IN MULTI-ITEM INVENTORY SYSTEMS

By

UMESH CHANDRA RAY

INDUSTRIAL & MANAGEMENT ENGINEERING PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JULY, 1980

COORDINATED REPLENISHMENT POLICIES IN MULTI-ITEM INVENTORY SYSTEMS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By
UMESH CHANDRA RAY

to the
INDUSTRIAL & MANAGEMENT ENGINEERING PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JULY, 1980

C E R T I F C A T E

This is to certify that the present work on "Coordinated Replenishment Policies in Multi-Item Inventory Systems", by Umesh Chandra Ray, has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

(Kripa Shanker)
Assistant Professor
Industrial and Management Engineering Programme
Indian Institute of Technology, Kanpur

July, 1980.

ACKNOWLEDGEMENTS

I express my deep sense of gratitude to my thesis supervisor Dr. Kripa Shanker who not only has provided to me his invaluable suggestions, undescribable help and unforgettable attachment but also has imbided in myself an extensive spectrum of academic rigour during the course of this work.

I acknowledge the extreme utility and importance of many informal discussions with and valuable advices of Dr. J.L. Batra which I had during my stay here.

It is indeed my pleasure to thank all my friends at I.I.T., Kanpur, whose amusing company made my two years of life in the campus a pleasant and memorable one.

I would like to thank Mr. J.K. Misra for his neat and accurate typing and Mr. Buddhi Ram Kandiyal for his immaculate cyclostyling work.

UMESH CHANDRA RAY

CONTENTS

Chapter		Page

ABSTRACT

Most of the real life inventory systems deal with stocking and issuing of many different items. The items interact among each other through sharing a common set-up cost of procurement or through competing for a given budget for inventory investment. The present work deals with the determination of optimal coordinated procurement policy parameters considering the above interactions in a multi-item inventory system.

Deterministic models have been developed for the situations allowing shortages and for the situations where group discounts on purchase are available. The joint replenishment models for stochastic demand case are developed for periodic review, order-up-to (R, T) and for continuous review reorder level, order quantity (r, Q) policies.

The solution methodology has been suggested in all cases and illustrations presented through numerical examples. Scopes for further work have been indicated at the end.

CHAPTER I

INTRODUCTION

1.1 Problem of Inventory Control - An Overview:

Inventories tie up most valuable asset; money. We may look on an inventory as a necessary evil even though it is listed in the accounts of the firm as an asset. Like all asset it ties up capital which can be put to other uses. No company has access to unlimited capital. With most, capital is definitely limited. New facilities, replacement facilities, raw materials, finished products, credit to customers, and other demands all complete for the available supply. The main objective of any business organisation is to make money. To quote Hoffman [21], 'Businesses exist to make money-money for the customer, money for the employee and money for the owner or stock holder. To continue making money, management must constantly increase its productivity through the effective utilization of materials, machines, man power and dollars .

There is a way to save money and generate additional capital at the same time. It will improve company's return on investment, and also make customer happier. The way is through better inventory management, which in turn can be defined as the sum total of those activities necessary for the acquisition, storage, sale, disposal, or use of material.

Since well managed inventories help reduce both the direct expenses of a business plus its investment base (and keep customers happier too), the business return on investment can be dramatically improved. Return on investment is the key ratio of : 'profits-to-investment', it is a prime indicator of the health of a business. Vigilant inventory management, by simultaneously improving profits and reducing investment, can make the business more healthy.

Careful management of inventory is a necessary in order to have adequate stockpiles of raw materials, supplies of ~~and~~ finished goods to provide the desired level of customer service, and no more. Inventory demands on capital will then be held to the lowest practical value.

Inventories are controllable items, which means we have an unexcelled opportunity to do something about the impact of inventories on costs, capital, investment, customer service, profits and return on the business investment.

1.2 Multi-Item Inventory System:

Although literature on inventory can be traced back to 1915, useful work has ensued only after 1951. The later work attempts at structuring real life systems. The formulation, however, has been so complex and cumbersome that most of the research workers have concentrated on only single product inventory models.

On the other hand if an inventory model is to have a practical significance, it should incorporate multi-stage, multi-product and multi-location aspects of inventory theory. Some efforts have been done to the study of individual areas mentioned above, but the field is still wide open for investigation. Once these areas are completely analysed, a real life situation-an integrated whole of the above - can be structured. Out of the three indicated, the area of multi-item models remains relatively untenanted. Hence, an attempt has been made here to make a practical incision into the field of multi-item-inventory models.

In many real life inventory and production situations, a family of items share a common supplier and/or a common production facility, a common storage space, a common budget, and so on. A common problem with such situations is to coordinate the replenishment or the production of the various items. We shall concentrate on the fixed set up cost of procurement for an inventory system. The results with little modifications apply for the production system. Typically there is a major set-up cost (S) associated with a replenishment of the family. In the procurement context this is the fixed (or header) cost of placing an order, independent of the number of distinct items involved in the order. In the production environment this is the change-over cost associated with converting the facility from the production

of some other family to production within the family of interest. Then there is a minor set-up cost (s_i) associated with including item i in a replenishment of the family. In the procurement context s_i is often called the line cost the cost of adding one more item or line to the requisition. From a production stand point s_i represents the relatively minor cost of switching to production of item i from production of some other item within the same family.

1.3 Literature Review:

The vast literature on inventory addresses the single item inventory systems. In the recent past, there has been an appreciable attempt to study the multi-item inventory systems. The problem of joint replenishment has been investigated by a number of authors. Each has discussed the problem under appropriate operating policy. Solution method which uses the dynamic programming has been suggested by Bomberger [2] Iterative solution procedures have been given by a number of authors namely Brown [3], Shu [19], Naddor [11], Doll and Whybark [5] and Goyal [6]. Nucturne [12] has given a graphical solution for the special case of two items. Solution procedures suggested by above authors, however, do not guarantee optimality. The drawback of above methods is that they all are iterative in nature, hence it is difficult to use on mannual basis. Silver [17] has presented a much

simpler non-iterative approach. He has considered a deterministic inventory model when the items are replenished jointly. It is extremely simple to use. The lot size reorder point model has been discussed at length by Hadley and Whitin [8]. Das [4] has developed some aids for lot-size inventory control under normal lead time demand. He has developed an explicit relationship between the two types of measures viz. one based on the penalty cost for shortage and the other on the probability of shortage. Approximations and reduction in dimensionality are the basic tools used for this purpose. These tools have previously been applied to this model by Herron [9], Parker [13], and Presutti and Trepp [15]. Comparable studies for the periodic review model are also reported by Sivazlian [18] and Snyder [20].

1.4 Organization of the Thesis:

The present study deals with analysis of a multi-item single location inventory system. The problem of multi-item coordinated replenishment with shortages backlogged has been investigated in Chapter II. Chapter III deals with coordinated replenishment with quantity discounts, on purchase of items. The case of coordinated replenishment stochastic demand - Periodic Review order-up - To (R,T) policy has been discussed in Chapter IV. In Chapter V, coordinated replenishment stochastic demand- continuous

Review Reorder level, order quantity (r, Q) policy has been discussed. Chapter VI offers some concluding remarks and puts forward some suggestion for further work.

CHAPTER II

DETERMINISTIC MODEL WITH SHORTAGES

2.1 Assumptions:

The assumptions made in the analysis of present system are essentially those prevalent in the derivation of classical EOQ except that now items are coordinated to reduce set-up costs.

i) The demand rate of each item is constant and deterministic.

ii) The unit variable cost of any of the items does not depend upon the replenishment quantity, in particular there are no quantity discounts.

iii) The replenishment lead time is constant.

iv) Shortages are backlogged.

v) The entire order quantity is delivered at the same time.

2.2 Notations:

D_i the usage rate of item i, in units/year.

v_i the unit variable cost of item i, in Rs./unit.

r the inventory carrying charge, in Rs./Rs./year.

Q_i the replenishment quantity of item i, in units.

S the major set-up (header) cost associated with a replenishment of a group involving one or more items, in Rs.

s_i the minor set-up (line) cost associated with item i which is included in the group under consideration in, Rs.

T the time interval between replenishments of the group, in years.

n the number of items in the group.

i item number ($i = 1,2,\ldots, n$).

K_i an integer, is the number of T intervals that the replenishment quantity of item i will last.

π_i back-order cost for item i, in Rs./unit short/year

C_H the total holding costs, in Rs./year.

C_B the total backorder costs, in Rs./year.

C_S the total set-up costs, in Rs./year.

TRC the total relevant costs, in Rs./year.

2.3 Problem Formulation and Solution Methodology:

Silver [17] has considered the situation of a family of items sharing a common supplier or common production facility. In his model he has considered two cost terms viz. set-up cost and carrying cost. There is a major set-up cost to make a replenishment of the family of items and a minor cost for each item included in the family. For a family of

items sharing a common supplier or common production facility, it makes sense to replenish the various items of a family jointly. He has developed his model assuming demand is deterministic, unit variable cost of the item is constant and shortages are not backlogged. He has solved the model for the best values of T (time between replenishments of the group and K_i's (the number of integer multiples of T that a replenishment of item i will last) which in turn when substituted in cost equation+ gives minimum total relevant cost.

The steps involved in Silver's model are summarized below for a quicker expositions.

Step 1:

Number the items such that

$\frac{s_i}{D_i v_i}$ is smallest for item 1. Set $K_1 = 1$.

Step 2:

Evaluate,

$K_i = [\frac{s_i}{D_i v_i} \cdot \frac{D_1 v_1}{S + s_1}]^{\frac{1}{2}}$ rounded to the nearest integer greater than zero.

Step 3:

Evaluate T^+ with K_i's found in Step 2. From

$$T^+ = [2 (S + \sum_{i=1}^{n} \frac{s_i}{K_i}) / r \sum_{i=1}^{n} K_i D_i v_i]^{\frac{1}{2}}$$

+ $$TRC (T, K_i's) = \frac{DTr}{2} \sum_{i=1}^{n} K_i D_i v_i + (S + \sum_{i=1}^{n} \frac{s_i}{K_i})/T$$

Step 4:

Determine,

$$Q_i v_i = K_i D_i v_i T^+ \quad , \qquad i = 1,2,\ldots, n$$

Goyal and Belton [7] have pointed out that Step 1 above be modified to number the items such that $\frac{S + s_i}{D_i v_i}$ rather than $\frac{s_i}{D_i v_i}$ is smallest for item 1. They have shown the modification to yield better results than Silver's original model.

Now we will proceed to formulate our problem of coordinated replenishments with shortages backlogged.

System Annual Cost:

Referring to Fig. 1, we see that if item i is replenished in a quantity sufficient to last for $K_i T$ years, where T is the time interval between replenishments of the family, then this quantity is given by,

$$Q_i = D_i K_i T$$

Also,

$$T_2 = b_i/D_i$$

$$T_1 = K_i T - T_2 = K_i T - b_i/D_i$$

$$= \frac{D_i K_i T - b_i}{D_i} = (Q_i - b_i)/D_i$$

We now derive different components of the annual cost.

a) Average Annual Set-up Cost:

Total set-up costs per year is given by,

$$C_S = \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{K_i T}$$

b) Average Annual Holding Cost:

$$C_H = r \sum_{i=1}^{n} \frac{v_i}{K_i \; T} \times \frac{(Q_i - b_i)^2}{2D_i}$$

c) Average Annual Backorder Cost:

$$C_B = \sum_{i=1}^{n} \frac{\pi_i}{K_i \; T} \times \frac{b_i^{\,2}}{2D_i}$$

Therefore, Total Relevant Cost

$$\begin{aligned}
TRC &= C_S + C_B + C_H \\
&= \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{K_i T} + \sum_{i=1}^{n} \frac{\pi_i \; b_i^{\,2}}{2D_i \; K_i T} \\
&\quad + r \sum_{i=1}^{n} \frac{v_i \; (D_i K_i T - b_i)^2}{2D_i \; K_i T} \\
&= \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{K_i T} + \sum_{i=1}^{n} \frac{b_i^{\,2}(\pi_i + rv_i)}{2D_i \; K_i T} \\
&\quad + r \sum_{i=1}^{n} \frac{v_i \; D_i \; K_i T}{2} - r \sum_{i=1}^{n} v_i \quad i
\end{aligned} \tag{1}$$

Setting,

$$\frac{\partial TRC}{\partial T} = 0, \qquad \frac{\partial TRC}{\partial b_i} = 0$$

gives the best T and b_i's for particular set of K_i's,

that is,

$$T^+ (K_i\text{'s}, b_i\text{'s}) = \Big[\frac{2}{r \sum_{i=1}^{n} v_i \;\; D_i K_i} \Big(S + \sum_{i=1}^{n} \frac{s_i}{K_i} + \sum_{i=1}^{n} \frac{b_i^{\,2} \; (\pi_i + rv_i)}{2D_i \; K_i} \Big) \Big]^{\frac{1}{2}} \tag{2}$$

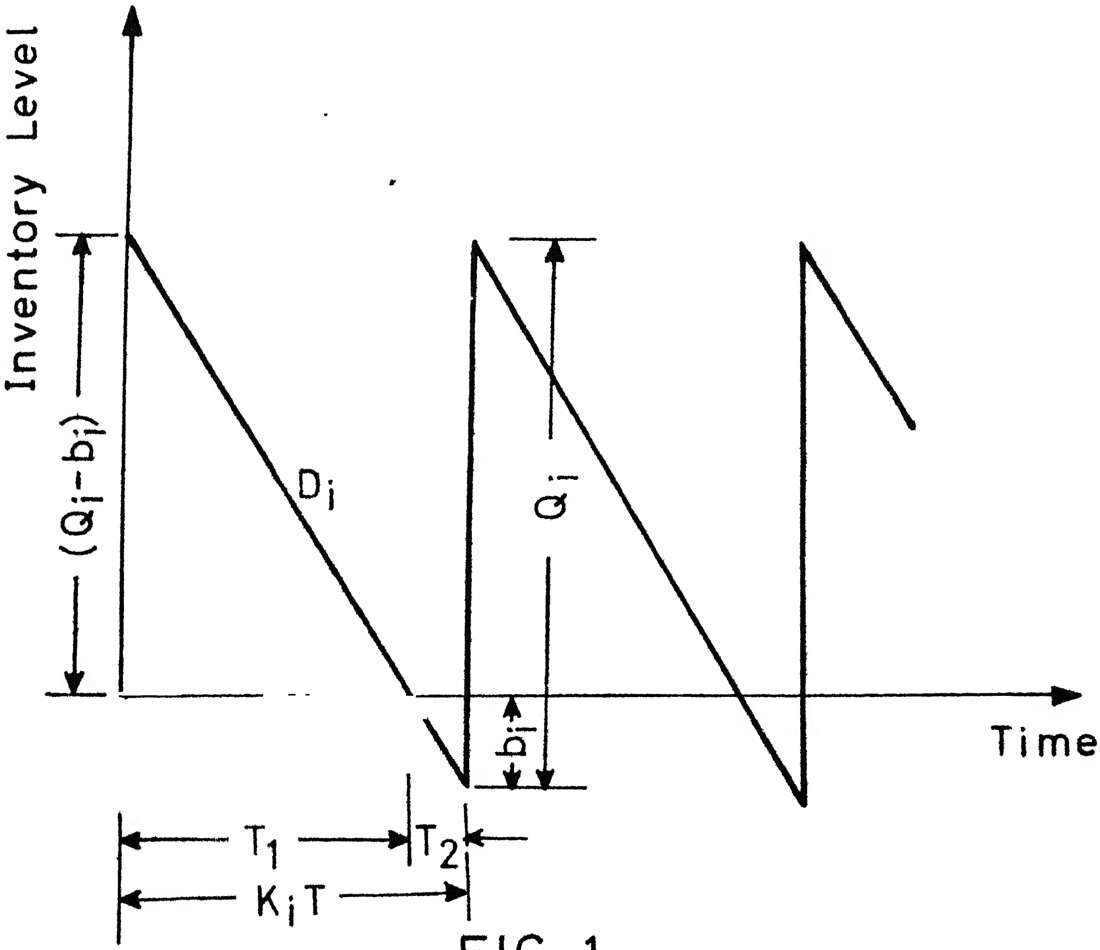

FIG. 1

JOINT REPLENISHMENTS WITH SHORTAGES BACKLOGGE

$$b_i^+ (K_i\text{'s}, T) = \frac{r\, v_i\, D_i\, K_i\, T}{(\pi_i + r\, v_i)} \tag{3}$$

From Eq. (2) after putting the value of b_i from Eq. (3).

$$T^+(K_i\text{'s}) = [2(S + \sum_{i=1}^{n} \frac{s_i}{K_i})/(r \sum_{i=1}^{n} v_i\, D_i\, K_i - r^2 \sum_{i=1}^{n} \frac{v_i^2\, D_i\, K_i}{(\pi_i + r v_i)})]^{\frac{1}{2}} \tag{4}$$

From Eqn. (3) and (4),

$$b_i^+ (K_i\text{'s}) = \frac{r\, v_i\, D_i\, K_i}{(\pi_i + r\, v_i)} [2(S + \sum_{i=1}^{n} \frac{s_i}{K_i})/(r \sum_{i=1}^{n} v_i\, D_i K_i - r^2 \sum_{i=1}^{n} \frac{v_i^2\, D_i\, K_i}{(\pi_i + r v_i)})]^{\frac{1}{2}} \tag{5}$$

After putting the values of T and b_i's from Eqs. (4) and (5) in Eq. (1) we have,

$$TRC^+ (K_i\text{'s}) = [2(S + \sum_{i=1}^{n} \frac{s_i}{K_i})(r \sum_{i=1}^{n} v_i\, D_i K_i - r^2 \sum_{i=1}^{n} \frac{v_i^2\, D_i\, K_i}{(\pi_i + r v_i)})]^{\frac{1}{2}} \tag{6}$$

Now our problem is to select the K_i's to minimize TRC^+ (K_i's). To minimize TRC^+ (K_i's) is equivalent to minimize

$$F(K_i\text{'s}) = [S + \sum_{i=1}^{n} \frac{s_i}{K_i}][r \sum_{i=1}^{n} v_i\, D_i\, K_i - r \sum_{i=1}^{n} \frac{v_i^2\, D_i K_i}{(\pi_i + r v_i)}] \tag{7}$$

If we ignore the integer constraints of K_i's and set partial derivatives of $F(K_i$'s$)$ equal to zero, then

$$\frac{F(K_i\text{'s})}{K_j} = 0 = \left[S + \sum_{i=1}^{n} \frac{s_i}{K_i}\right]\left[r v_j D_j - r^2 \frac{v_j^2 D_j}{(\pi_j + r v_j)}\right]$$

$$- \frac{s_j}{K_j^2}\left[r \sum_{i=1}^{n} v_i D_i K_i - r^2 \sum_{i=1}^{n} \frac{v_i^2 D_i K_i}{(\pi_i + r v_i)}\right]$$

or,

$$K_j^2 = \frac{s_j\left[\sum_{i=1}^{n} v_i D_i K_i - r \sum_{i=1}^{n} \frac{v_i^2 D_i K_i}{(\pi_i + r v_i)}\right]}{v_j D_j \left[S + \sum_{i=1}^{n} \frac{s_i}{K_i}\right]\left[\frac{\pi_j}{\pi_j + v_j r}\right]}, \quad j = 1, 2, \ldots, n$$

Put,

$$\frac{\pi_j}{\pi_j + r v_j} = E_j$$

$$K_j^2 = \frac{s_j}{v_j D_j E_j}\left[\frac{\sum_{i=1}^{n} v_i D_i K_i - r \sum_{i=1}^{n} \frac{v_i^2 D_i K_i}{(\pi_i + r v_i)}}{S + \sum_{i=1}^{n} \frac{s_i}{K_i}}\right] \quad (8)$$

For $j \neq m$ we have,

$$\frac{K_j^2}{K_m^2} = \frac{s_j}{v_j D_j E_j} \times \frac{v_m D_m E_m}{s_m}$$

or,

$$\frac{K_j}{K_m} = \left[\frac{s_j}{v_j D_j E_j} \times \frac{v_m D_m E_m}{s_m}\right]^{\frac{1}{2}}$$

It is seen that if

$$\frac{s_j}{v_j D_j E_j} < \frac{s_m}{v_m D_m E_m}$$

then (the continuous solution) K_j is less than (the continuous solution)K_m Therefore, the item i for which the ratio $s_i/(v_i\ D_i\ E_i)$ is smallest will have the lowest value of K_i, namely 1. If the items are numbered such that the item 1 has the smallest value of $s_i/(v_i\ D_i\ E_i)$ then from above,

$$K_1 = 1$$

From Eqn. (8),

$$K_j = C\left[\frac{s_j}{v_j\ D_j\ E_j}\right]^{\frac{1}{2}}, \qquad j = 2,3,\ldots, n$$

where,

$$C = \left[\left(\frac{\sum_{i=1}^{n} v_i\ D_i\ K_i - r\sum_{i=1}^{n}\frac{v_i^2\ D_i\ K_i}{(\pi_i + r v_i)}}{S + \sum_{i=1}^{n}\frac{s_i}{K_i}}\right)\right]^{\frac{1}{2}} \tag{9}$$

Now,

$$\sum_{i=1}^{n} v_i\ D_i\ K_i = v_1\ D_1 + C\sum_{i=2}^{n} v_i\ D_i\left[\frac{s_i}{v_i\ D_i\ E_i}\right]^{\frac{1}{2}}$$

$$= v_1\ D_1 + C\sum_{i=2}^{n}\left[\frac{s_i\ v_i\ D_i}{E_i}\right]^{\frac{1}{2}}$$

$$\sum_{i=1}^{n}\frac{v_i^2\ D_i\ K_i}{(\pi_i + r v_i)} = \frac{v_1^2\ D_1}{(\pi_1 + r v_1)} + C\sum_{i=2}^{n}\frac{v_i^2\ D_i}{(\pi_i + v_i r)}\left[\frac{s_i}{v_i\ D_i\ E_i}\right]^{\frac{1}{2}}$$

$$= \frac{v_1^2\ D_1}{\pi_1 + v_1 r} + C\sum_{i=2}^{n}\left[\frac{s_i\ v_i^3 D_i}{\pi_i(\pi_i + v_i r)}\right]^{\frac{1}{2}}$$

$$\sum_{i=1}^{n} \frac{s_i}{K_i} = s_1 + \frac{1}{C} \sum_{i=2}^{n} s_i/[s_i/(v_i D_i E_i)]^{\frac{1}{2}}$$

$$= s_1 + \frac{1}{C} \sum_{i=2}^{n} [s_i \; v_i \; D_i \; E_i]^{\frac{1}{2}}$$

Putting these values in Eqn. (9), we have,

$$C = [\frac{v_1 \; D_1 \; E_1}{S + s_1}]^{\frac{1}{2}}$$

Hence,

$$K_j = [\frac{s_j}{v_j \; D_j \; E_j} \; \frac{v_1 D_1 E_1}{S + s_1}]^{\frac{1}{2}} \quad \text{for } j = 2,3,\ldots, n \qquad (10)$$

The values of K_j ($j = 2,3,\ldots, n$) is rounded to the nearest integer greater than zero.

Similar to the steps suggested by Silver [17] we have the following steps for obtaining the K_i's, T, c_i's, TRC and b_i's.

Step 1:

Items are numbered such that,

$$s_i / \; v_i \; D_i \; E_i$$

is smallest for item 1. Set $K_1 = 1$.

Step 2:

Evaluate,

$$K_i = [\frac{s_i}{v_i \; D_i \; E_i} \times \frac{v_1 \; D_1 \; E_1}{S + s_1}]^{\frac{1}{2}}$$

Round off the above values of K_i's to the nearest integer greater than zero.

Step 3: Evaluate TRC^+ (K_i's) from Eq. (6) using K_i's from Step 2.

Step 4: Evaluate T^+ (K_i's) from Eq. (4) using K_i's from Step 2.

Step 5: Evaluate $Q_i = K_i \; D_i \; T$

Step 6: Evaluate b_i^+ (K_i's) from Eq. (5) using K_i's from Step 2.

A Graphical Aid:

If we have n items in a group to be replenished then from Eq. (10) we see that we will have to take the square root of Eq. (10) (n-1) times to get the different values of K_i's. To avoid this square root business, in turn to reduce the computational complexities, a graphical approach is presented here. Because of the rounding rule of Eq. (10) to get the integer values of K_i's we are indifferent between the integer values K_i and K_i+1 when

$$\left[\frac{s_i}{v_i \; D_i \; E_i} \times \frac{v_1 \; D_1 \; E_1}{S + s_1} \right]^{\frac{1}{2}}$$

is half way between K_i and K_i+1, that is

$$K_i + \frac{1}{2} = \left[\frac{s_i}{v_i \; D_i \; E_i} \quad \frac{v_1 \; D_1 \; E_1}{S + s_1} \right]^{\frac{1}{2}}$$

or,

$$\frac{s_i}{v_i\ D_i\ E_i} = (K_i + \frac{1}{2})^2 \frac{S + s_1}{v_1\ D_1\ E_1}$$

Thus the indifference curves are straight lines passing through the origin in a plot of $s_i/(v_i D_i E_i)$ vs. $S + s_1/(v_1 D_1 E_1)$ as shown in Fig.2. Multiplication of $s_i/(v_i D_i E_i)$ and $S+s_1/(v_1 D_1 E_1)$ by the same constant does not have any affect on Eq. (10). This flexibility helps us in the plot of graph. Suppose both the ratios are extremely small, then multiplying both by the same large number, we can ascertain the corresponding points on the graph with great ease.

2.4 Numerical Illustration:

Consider a family of 5 items with the following characteristics –

S = Rs. 10 $\qquad$ r = 0.2 Rs./Rs./year

Item	s_i	π_i	v_i	D_i
1	1.00	6	0.50	10,000
2	4.00	5	0.40	1,000
3	6.00	4	0.35	12,000
4	8.00	3	0.20	500
5	9.00	2	0.15	400

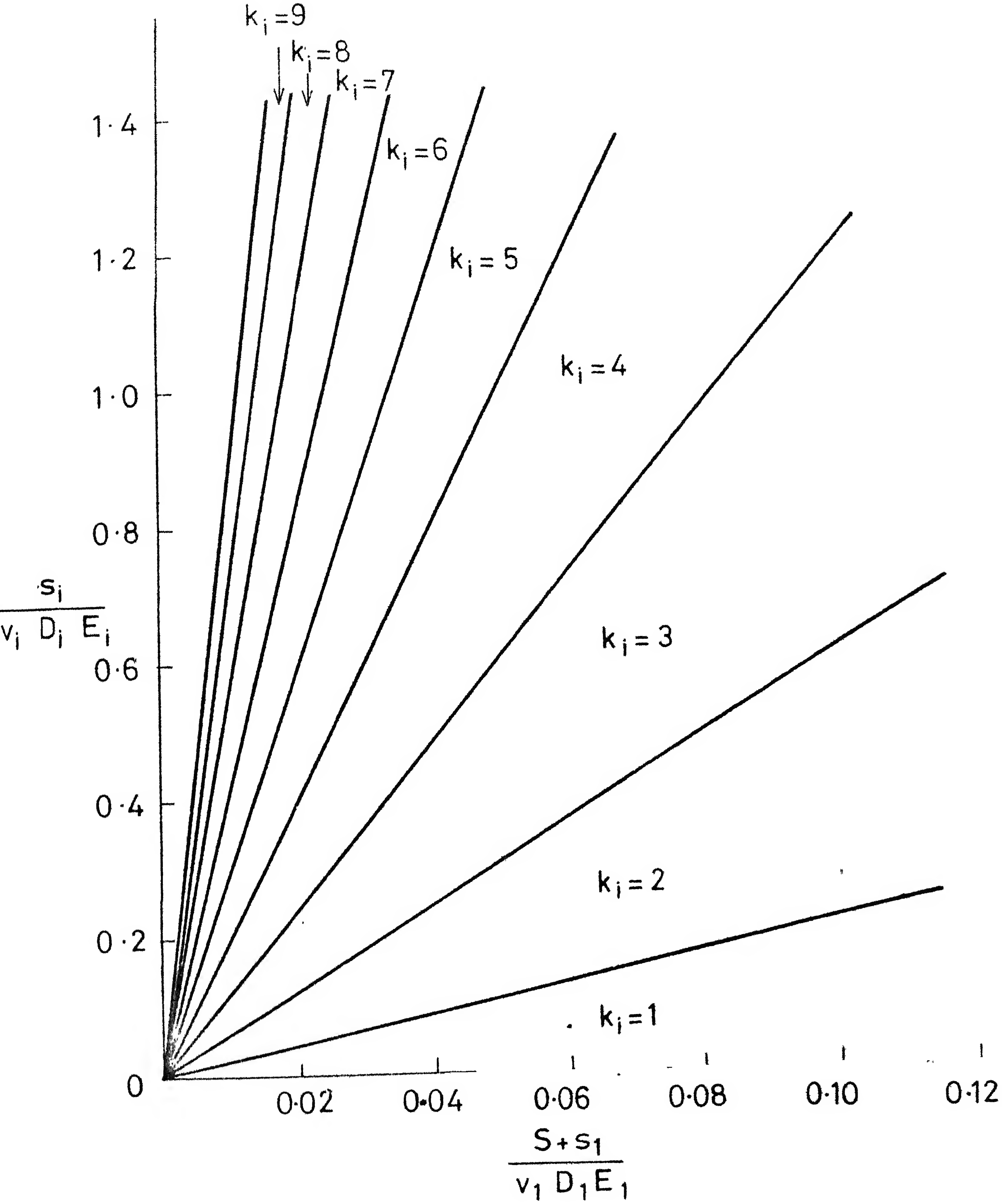

FIG. 2

IMPLEMENTATION AID FOR THE DETERMINATION OF THE Ki's

A computerized version of the method is developed in FORTRAN 10 and control parameters have been calculated on DEC system 1090. The results are as follows:

T(YR) = Time between replenishment = 0.139

TRC(Rs./YR) = Total Relevant Cost = 308.06

Above values have been compared with values when K_i's for all the items have been taken as 1. i.e. items are treated independently. Comparing these two values of total relevant costs, we see that total relevant cost is more when all K_i's are 1. Hence, it is economical to coordinate the items in a group.

Compared results for the illustrative example:

Method	K_i's					T (Yrs.)	TRC (Rs./Yr.)
	K_1	K_2	K_3	K_4	K_5		
This Method	1	2	1	6	8	0.139	308.06
All K_i =	1	1	1	1	1	0.197	384.89

CHAPTER III

DETERMINISTIC MODEL WITH QUANTITY DISCOUNTS

3.1 Statement of the Problem:

Quantity discounts (All units discount) may be offered on the total volume of a replenishment made up of several items. Here in our problem items are replenished jointly so that most of the times total volume replenished will be sufficient to ask for quantity discounts. As in the case of single item if discount is taken into account the replenishment costs (both fixed and unit costs) will decrease but at the same time inventory carrying costs will increase. For multiple items if every item is included in every replenishments, the analysis will be like that of single item. But our case is different. Here every item is not included in every replenishment. Items for which $K_i > 1$ is not included in every replenishment even to help achieve a quantity discount level. So on certain replenishments a discount is achieved while on others it is not. Replenishment cycles would be no longer of the same duration T (the ones where quantity discounts are achieved will be longer than the others).

As done by Silver [17] consider three possible solutions. The first is one where coordinated analysis is done assuming discount. If the replenishment quantities are sufficient to achieve discount then we are through otherwise not. The second case is where the best result is achieved right at the break point. The third solution is the coordinated solution without a quantity discount. The last two cases are compared. K_i's and T for which the total relevant cost is minimum is used.

3.2 Assumptions:

We make the following assumptions to further characterize the system:

a) Planning horizon is finite

b) Demand rate is deterministic and static

c) Shortages are allowed

d) Holding cost is linear in nature

3.3 Nomenclature:

We use the following notations:

v_{oi} = unit cost of item i without a discount.

v_i = unit cost of item i with a discount,

d = discount rate,

Q_b = break point quantity,

Q_{sm} = summation of order quantities for all items having $K_i = 1$.

K_i = The number of integer multiples of T that a replenishment of item i will last,

Q_i = The replenishment quantity of item i

D_i = The demand rate of item i,

S = The major set-up cost, associated with the replenishment of the group,

s_i = The minor set-up cost, incurred when item i is included in the group.

3.4 Problem Formulation and Solution Methodology:

Following are the four steps to get the values of Q_i's considering quantity discounts:

Step 1:

Expressions for K_i's and T are developed as in Sec.2.3, but with the difference that now each $v_i = v_{oi}(1-d)$. Now the size of the smallest family replenishment is computed which is equal to the summation of order quantities of all items for which $K_i = 1$. If $Q_{sm} \geq Q_b$, the values of K_i's, T and Q_i's found above are optimal. If $Q_{sm} < Q_b$, we proceed to Step 2.

Step 2:

Now the family cycle time T is set equal to T_b which in turn is defined as

T_b = Q_b/(Summation of D_i's of all items having K_i = 1).

According to Eq. (1) of Sec. 2.3, the cost expression at this break point will be as follows:

$$\text{TRC}\ (T_b,\ K_i\text{'s}) = (1-d)\sum_{i=1}^{n} D_i\, v_{oi} + \frac{S + \sum_{i=1}^{n} \frac{s_i}{K_i}}{T_b}$$

$$+ \frac{r(1-d)}{2T_b}\sum_{i=1}^{n} \frac{v_{oi}(Q_i - b_i)^2}{D_i\ K_i} + \frac{1}{2T_b}\sum_{i=1}^{n} \frac{\pi_i\ b_i^2}{D_i\ K_i}$$

Step 3:

Best values of K_i's, T and Q_i's are found out as in Sec. 2.3 without quantity discount. According to Eq. (6) of Sec. 2.3 the total relevant cost of this solution will be given as,

$$\text{TRC (best T and } K_i\text{'s)} = \sum_{i=1}^{n} D_i\ v_{oi}$$

$$+ [2(S + \sum_{i=1}^{n} \frac{s_i}{K_i})(r \sum_{i=1}^{n} v_{oi}\ D_i K_i - r^2 \sum_{i=1}^{n} \frac{v_{oi}^2\ D_i K_i}{\pi_i + r v_{oi}})]^{\frac{1}{2}}$$

Step 4:

Cost of the best solution without a discount and the cost of the solution where the smallest replenishment quantity is right at the break point are compared i.e. TRC values of Step 2 and Step 3 are compared, whichever of these has the lower cost is then the solution to use. The K_i's, T and Q_i's associated with this solution are to be used.

3.5 Numerical Example:

Consider a family of 5 items with following characteristics:

$S = 10$ $\qquad$ $C_b = 3650$

s_i	D_i	v_i	H_i	π_i
4.00	543	20	5	6
5.89	350	24	6	5
7.94	250	28	7	4
8.19	110	32	8	3
8.87	100	36	9	2

A computerized version of the method is developed in FORTRAN 10 and decision variable have been calculated on DEC System 1090. For a particular value of discount optimal results are as follows:

Discount = 0.05

T (YR) = Time between replenishments = 0.1390

TRC (T_b, K_i's) = Total relevant costs (RS/YR) = 32234.192

Items	1	2	3	4	5
K_i's	1	1	1	2	2
Q_i's	75	49	35	31	28

Thus for discount of 0.05 it is optimal to replenish items 1, 2 and 3 after every 0.1390 yrs. and items 4 and 5 after every 0.2780 years.

Similarly for different value of discount we can

CHAPTER IV.

STOCHASTIC MODEL - PERIODIC REVIEW ORDER-UP-TO (R, T) POLICY

4.1 Statement of the Problem:

Here the decision rules for coordinated replenishment under stochastic demands will be developed for periodic review system of order-up-to-type. The logic of the basic single item (R, T) model has been extended to include multi-item case. In (R, T) model after every T units of time a quantity is ordered to raise the inventory position to R_i for each item. The analysis of the problem is same as for single item except that here items are replenished jointly to reduce the set-up cost.

4.2 Assumptions:

a) The inventory levels are reviewed every T units of time and replenishment decisions can be made only at those times.

b) The unit cost C of the item is constant, independent of the quantity ordered.

c) The cost of each backorder is π and the cost is independent of length of time for which the backorder exists.

d) Backorders are incurred only in very small quantities

so that when order arrives, it is almost always sufficient to meet any outstanding backorders.

e) The replenishment lead time is of fixed duration.

f) The demand for each item i in a period of duration $(\tau + T)$ is normally distributed with known mean $\bar{x}_{\tau+T,i}$ and standard deviation $\sigma_{\tau+T,i}$.

4.3 Nomenclature:

S = Fixed set-up cost of replenishment of family of items including review cost,

D_i = Demand rate of item i,

s_i = Variable set-up cost when item i is included in a group to be replenished,

K_i = The number of integer multiples of T that a replenishment of item i will last,

T = Time interval between reviews,

H_i = Holding cost per unit per year of item i,

R_i = Order quantity of item i,

μ_i = Mean demand during lead time of item i,

σ_i = Standard deviation of demand during lead time of item i,

π_i = Penalty cost per unit short,

n = Number of items in a group.

4.4 Problem Formulation and Solution Methodology:

Under the assumed policy a group replenishment is made every T years and item i is replenished in a quantity sufficient to last for $K_i T$ years. Therefore,

$$Q_i = D_i K_i T \qquad (i = 1, 2, \ldots, n)$$

The various costs shall be given as follows:

a) Average annual set-up cost

$$C_s = \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{K_i T}$$

b) Average annual cost of back-order.

Expected back-order during a cycle

$$\bar{B}(R_i, K_i T) = \int_{R_i}^{\infty} (x_i - R_i) f(x_i, \tau + K_i T) \, dx_i$$

Define $\hat{l}(x_i, K_i T) = f(x_i, \tau + K_i T)$.

Therefore,

$$\bar{B}(R_i, K_i T) = \int_{R_i}^{\infty} (x_i - R_i) \hat{l}(x_i, K_i T) \, dx_i$$

The average number of backorders incurred per year,

$$E(R_i, K_i T) = \frac{1}{K_i T} \int_{R_i}^{\infty} (x_i - R_i) \hat{l}(x_i, K_i T) \, dx_i$$

Hence average annual cost of backorders,

$$C_B = \sum_{i=1}^{n} \frac{\pi_i}{K_i T} \int_{R_i}^{\infty} (x_i - R_i) \hat{l}(x_i, K_i T) \, dx_i$$

c) Average annual cost of holding inventory:

Expected net inventory just after the receipt of an order:

$$= R_i - \mu_i$$

Expected net inventory prior to receiving an order

$$= R_i - \mu_i - D_i K_i T$$

Now because of assumption 'd' it must be true that the integral over time of the net inventory must very closely approximate the integral over time of the on hand inventory. Hence the expected unit of storage incurred per period is to a good approximation,

$$[\tfrac{1}{2} (R_i - \mu_i) + \tfrac{1}{2} (R_i - \mu_i - D_i K_i T)] K_i T$$

$$= [R_i - \mu_i - \frac{D_i K_i T}{2}] K_i T$$

So that average annual cost of carrying inventory is,

$$C_H = \sum_{i=1}^{n} H_i (R_i - \mu_i - \frac{D_i K_i T}{2})$$

Hence, average annual variable cost is

$$C = C_s + C_H + C_B = \frac{S}{T} + \sum_{i=1}^{n} \frac{s_i}{K_i T} + \sum_{i=1}^{n} H_i (R_i - \mu_i - \frac{D_i K_i T}{2}$$

$$+ \sum_{i=1}^{n} \frac{\pi_i}{K_i T} \int_{R_i}^{\infty} (x_i - R_i) \hat{1} (x_i, K_i T) \, dx_i$$

Derivation of the Proposed Decision Rules:

For a given T and K_i's, setting $\partial C/\partial R_i = 0$ gives the best R_i for the particular set of K_i's.

$$\frac{\partial C}{\partial R_i} = 0 = H_i - \frac{\pi_i}{K_i T} \int_{R_i}^{\infty} \hat{l}\,(x_i,\ K_i T)\ dx_i$$

$$= H_i - \frac{\pi_i}{K_i T}\, L\,(R_i,\ K_i T)$$

where,

$$L\,(R_i,\ K_i T) = \int_{R_i}^{\infty} \hat{l}(x_i,\ K_i T)\ dx_i$$

Thus R_i^+, optimal value of R_i is a solution to

$$L(R_i,\ K_i T) = \frac{H_i\ K_i T}{\pi_i}$$

or
$$\left(\frac{R_i - \mu_i}{\sigma_i}\right) = \frac{H_i\ K_i T}{\pi_i}$$

or
$$R_i = \mu_i + \sigma_i\ Z_{p_i} \qquad \text{where } p_i = \frac{H_i\ K_i T}{\pi_i}$$

By giving different values to sets of K_i and T we can have different values of R_i. For each set of K_i's changing the value of T, the cost C for each T is calculated. The T corresponding to the minimum cost is chosen. The set of K_i's which gives lowest of these minimum is taken as

optimal one. Due to the probabilistic nature of demand to get an exact expression for K_i (the number of integer multiples of T that a replenishment of item i will last) is quite involved. So different sets of K_i's have been supplied as problem parameter and total relevant costs for each set are calculated and then compared to give that set of K_i's which gives minimum total relevant cost.

4.5 Numerical Example:

Consider a family of 5 items with following characteristics:

S = Rs. 10.00 $\qquad\qquad \tau$ = 0.03 yr.

s_i	π_i	H_i	D_i	μ_i	σ_i^2
3	25	3	50	75	100
4	20	4	400	500	600
5	24	3	100	150	200
6	23	2	150	200	250
7	21	5	500	600	650

A computerized version of the method is developed in FORTRAN 10 and decision variables have been calculated on DEC System 1090.

For different sets of K_i's, optimal values of time between replenishments and total relevant cost have been

calculated. Finally one having the lowest of these minimum is taken as optimal one. The results are as follows:

K_i's					T (Yrs.)	TRC (Rs./Yr.)
K_1	K_2	K_3	K_4	K_5		
1	1	1	1	1	0.10	1004.85
2	1	2	2	1	0.08	992.91+
3	1	2	2	1	0.08	1001.65
2	1	2	1	1	0.08	1004.00
2	1	1	1	1	0.08	1003.89

Lowest of these costs is 992.91 Rs./Yr. So our optimal policy is to set the values of K_i's as 2,1,2,2,1. One can also note that the total relevant cost when all K_i's are 1, is highest compared to other costs where items are jointly replenished. Hence it is beneficial to replenish the items jointly.

CHAPTER V

STOCHASTIC MODEL - CONTINUOUS REVIEW REORDER LEVEL, ORDER QUANTITY (r, Q) POLICY

5.1 Statement of the Problem:

One of the most popular types of inventory management is the (r, Q) system, in which a quantity Q of an item is reordered whenever the inventory position goes to the reorder point r or below. The use of (r, Q) inventory policy has grown rapidly as automatic data processing equipment has become available to perform the necessary calculations and transaction reporting. Computer firms have developed detailed package programmes for continuous transactions surveillance and calculation of order quantities and reorder points. Das [4] has developed a model which is suitable for computer application to determine accurately the minimum cost values of Q and r, employing either stockout penalties or specified service levels. In his model he has developed an equivalance relationship between the penalty cost and disservice level assuming that the demand during lead time is normally distributed. He has also developed explicit formulas for the optimal order size and reorder point. To

reduce the computational complexities he has transformed the equations into a single equation of one unknown.

Das has developed some aids for lot size inventory control considering single item and without any constraint. To make the problem more realistic we have extended Das's work to includ multi-item, budgetary constraints. The reorder level for different item is different. As soon as the item i crosses its reorder level r_i, an amount Q_i is ordered for that item. The problem has been worked for an optimal order quantity Q_i, its optimal reorder level r_i and an equivalence relationship between the penalty cost and disservice level. The model has been supported with numerical example also at the end of the Chapter.

5.2 Assumptions:

a) Out-of-stock items may be backordered and delivered when available,

b) Periods of stockouts are of short duration so that the average inventory can be approximated as the sum of the safety stock plus one half the order quantity.

c) The replenishment lead time is constant and the demand during lead time is normally distributed.

d) The unit cost C of an item is independent of Q i.e. quantity discount is not allowed.

c) There is never more than a single order outstanding for an item i.

5.3 Nomenclature:

C_i = Unit cost of item i,

D_i = Expected annual demand of item i,

Q_i = Order size of item i,

R_i = Reorder point of item i,

A_i = Fixed order cost per order for item i,

H_i = Holding cost per unit per year of item i,

π_i = Penalty cost per unit short of item i,

μ_i = Mean lead time demand of item i,

σ_i = Standard deviation of lead time demand of item i,

θ = Lagrangian multiplier,

B = Upper limit of the budget.

5.4 Problem Formulation and Solution Methodology:

The lot size reorder point inventory model has been discussed at length by Hadley and Whitin [8]. They have developed an expression for average annual cost for single item without any constraints. On the same line the average annual cost for multi-item with budgetary constraints is given by,

$$K(Q_i, R_i) = \sum_{i=1}^{n} \left[\frac{D_i A_i}{Q_i} + \left(\frac{Q_i}{2} + R_i - \mu_i\right) H_i + \frac{D_i \pi_i}{Q_i} L(R_i) \right] \qquad (1)$$

Such that $$\sum_{i=1}^{n} C_i Q_i \leq B \qquad (2)$$

where $$L(R_i) = \int_{R_i}^{\infty} (x_i - R_i)\, h(x_i)\, dx_i \qquad (3)$$

$h(x_i)$ being the probability density of lead time demand assumed to be normal with mean μ_i and standard deviation σ_i.

First we solve the problem ignoring the constraint (2), i.e. we minimize over each Q_i separately. If these Q_i's satisfy (2), then those Q_i's are optimal. In such a case constraint is not active.

On the other hand if the Q_i's do not satisfy (2) then the constraint is active and the Q_i's are not optimal. To find the optimal Q_i's and R_i's, the Lagrange multiplier technique is used. We form the function,

$$J = \sum_{i=1}^{n} \left[\frac{D_i A_i}{Q_i} + \left(\frac{Q_i}{2} + R_i - \mu_i\right) H_i + \frac{D_i \pi_i}{Q_i} L(R_i) \right] + \theta \left(\sum_{i=1}^{n} C_i Q_i - B \right) \qquad (4)$$

where the parameter θ is a Lagrange multiplier.

Then the set of Q_i's and R_i's, $i = 1, 2, \ldots, n$ which yield the absolute minimum of Eq.(1) subject to Eq.(2) are solution to the set of equations,

$$\frac{\partial J}{\partial Q_i} = -\frac{D_i A_i}{Q_i^2} + \frac{H_i}{2} - \frac{D_i \pi_i}{Q_i^2} L(R_i) + \theta C_i = 0 \qquad (5)$$

$$\frac{\partial J}{\partial \theta} = C_i Q_i - B = 0 \qquad (6)$$

$$\frac{\partial J}{\partial R_i} = H_i - \frac{D_i \pi_i}{Q_i} \int_{R_i}^{\infty} h(x_i)\, dx_i$$

or,

$$\int_{R_i}^{\infty} h(x_i)\, dx_i = \frac{H_i Q_i}{D_i \pi_i} \qquad (7)$$

From Eq. (5) we have,

$$Q_i^2 = [\, 2D_i/(H_i + 2\theta C_i)][A_i + \pi_i L(R_i)] \qquad (8)$$

Now let $f(.)$ be the standardized normal density and define,

$$M(r_i) = \int_{Z_{r_i}}^{\infty} (Z_i - Z_{r_i})\, f(Z_i)\, dZ_i$$

where Z_{r_i} is such that $r_i = \int_{Z_{r_i}}^{\infty} f(Z_i)\, dZ_i$

Let $p_i = \frac{H_i Q_i}{D_i \pi_i} = \int_{R_i}^{\infty} h(x_i)\, dx_i$

= risk of stockout of item i during lead time for given Q_i

$$Z_{p_i} = \frac{R_i - \mu_i}{\sigma_i} \quad \text{or} \quad R_i = \mu_i + \sigma_i Z_{p_i} \qquad (9)$$

Substituting Eq. (9) in Eq. (3) it can be shown that

$$L(R_i) = \sigma_i \, M(p_i)$$

Using above in Eq. (8) and simplifying

$$Q_i^2 = \frac{2D_i}{H_i + 2\Theta C_i} [A_i + \pi_i \sigma_i \, M(p_i)]$$

or

$$\frac{D_i \, \pi_i \, (H_i + 2\Theta C_i)}{2\sigma_i \, H_i^2} \, p_i^2 - \frac{A_i}{\pi_i \, \sigma_i} = M(p_i)$$

or,

$$u_i \, p_i^2 - v_i = M(p_i) \tag{10}$$

where, $u_i = \dfrac{D_i \, \pi_i \, (H_i + 2OC_i)}{2\sigma_i \, H_i^2}$, $v_i = \dfrac{A_i}{\pi_i \, \sigma_i}$

Thus, the original problem of determining Q_i^+ and R_1^+ is reduced to determining p_i^+ such that Eq. (10) is satisfied. There is no explicit expression for M here, therefore M is approximated by a quadratic form because L.H.S. of Eq.(10) is quadratic in p. Therefore, fitting $ap_i^2+bp_i+c$ (by least squares method) through 59 values of M in the range $0.004 \leq p_i \leq 0.499$, we find a = 0.79838, b = 0.39694, and c = -0.00044 with 0.001 (approx.) as the maximum absolute error.

Hence, p_i^+ can be estimated by the positive root of the eqn.

$$u_i p_i^2 - v_i = ap_i^2 + bp_i + c \tag{11}$$

or,

$$(u_i - a) \, p_i^2 - bp_i - (v_i + c) = 0$$

Therefore,

$$p_i^+ = \frac{b + [b^2 + 4(u_i - a)(v_i + c)]^{\frac{1}{2}}}{2(u_i - a)} \tag{12}$$

which gives,

$$Q_i^+ = \frac{D_i \pi_i}{H_i} p_i^+ \tag{13}$$

$$R_i^+ = \mu_i + \sigma_i Z_{p_i^+} \tag{14}$$

Further, multiplying both sides of Eq. (8) by $H_i + 2\theta C_i / 2C_i$ and rearranging we get,

$$\frac{D_i \pi_i}{C_i} L(R_i) = \frac{Q_i(H_i + 2\theta C_i)}{2} - \frac{A_i D_i}{Q_i} \tag{15}$$

Now from Eq. (1) and Eq. (15), we have,

$$K(Q_i^+, R_i^+) = \sum_{i=1}^{n} [(Q_i + R_i - \mu_i) H_i + \theta C_i]$$

$$= \sum_{i=1}^{n} [(Q_i + \sigma_i Z_{p_i^+}) H_i + \theta C_i] \tag{16}$$

Substituting the values of u_i and v_i in Eq. (10) and writing $M(p_i) = M_i$

$$\frac{D_i \pi_i (H_i + 2\theta C_i)}{2\sigma_i H_i^2} p_i^2 - \frac{A_i}{\pi_i \sigma_i} = M_i$$

or, $$\left(\frac{p_i^2}{2}\right)\left(\frac{D_i (H_i + 2\theta C_i)}{H_i^2}\right) \pi_i^2 - M_i \sigma_i \pi_i - A_i = 0$$

This is quadratic in π_i, hence for $\pi_i > 0$

$$\pi_i^{+} = \frac{M_i\ \sigma_i + [\ (M_i\sigma_i)^2 + 2A_i p_i^{\,2}\ (\frac{D_i(H_i+2\theta C_i)}{H_i^{\,2}})\,]^{\frac{1}{2}}}{p_i^{\,2}\ (\ \frac{D_i(H_i+2\theta C_i)}{H_i^{\,2}}\ )}$$

$$= \frac{H_i^{\,2}}{p_i D_i(H_i+2\theta C_i)}\ [\ \frac{M_i\sigma_i}{p_i} + ((\ \frac{M_i\sigma_i}{p_i})^2$$

$$+ \frac{2A_i D_i(H_i+2\theta C_i)}{H_i^{\,2}}\ )^{\frac{1}{2}}] \;=\; \pi_i^{+}\ (p_i, \sigma_i) \qquad (17)$$

Since $M(p_i) < p_i$ for all $p_i < 0.5$, hence as p_i increases r.h.s. of Eq. (17) decreases, consequently π_i^{+} decreases. Thus π_i^{+} and p_i are inversely related. This emplies that higher the penalty cost (π_i^{+}) greater is the protection against stockouts (p_i). Similarly an increase in the variability of lead time demand increases penalty cost and calls for higher level of service in the optimum.

If service level is reduced by increasing p_i to p_i', then the unit penalty cost must be reduced by an amount,

$$\Delta_{1,i} = \pi_i^{+}\ (p_i,\ \sigma_i) - \pi_i^{+}\ (p_i',\ \sigma_i)$$

Similarly, if the variance of lead time demand increases from $\sigma_i^{\,2}$ to $\sigma_i'^{\,2}$, then the unit penalty cost must be increased by,

$$\Delta_{2,i} = \pi_i^{+}\ (p_i,\ \sigma_i') - \pi_i^{+}\ (p_i, \sigma_i)$$

5.5 Numerical Example:

Consider a family of 6 items and assume that the following parameter values are known,

D_i	H_i	C_i	A_i	π_i	μ_i	σ_i
3430	14	20	6	1.0	350	60
2000	12	100	8	1.2	350	30
1091	11	50	9	1.1	100	15
1500	12	90	7	1.2	150	15
2700	13	40	5	1.0	250	20
4000	15	60	8	1.1	400	30

A computerised version of the method is developed and decision variables have been calculated on DEC system 1090. By changing the value of θ, grand total relevant costs are calculated. It has been seen that as θ increases, the grand total relevant cost first decreases to give minimum and then increases. θ corresponding to minimum cost is taken as optimal one. The various results are as follows:

θ (Theta)	GTRC (Rs./Yr)	θ(Theta)	GTRC (Rs./Yr)
0.00	6454.00	4.00	4415.60
0.50	4510.85	4.50	4459.25
1.00	4352.90	5.00	4462.40
1.50	4327.80+	5.50	4477.30
2.00	4339.00	6.00	4511.55
2.50	4339.10	6.50	4539.25
3.00	4371.45	7.00	4550.00
3.50	4389.65		

From above results, we see that GTRC corresponding to $\theta = 1.5$ is the lowest. Hence this is taken as optimal one.

Corresponding to $\theta = 1.5$, the following table gives the various values of Q_i^+'s, R_i^+'s, total cost and p_i^+'s.

Item	Q_i^+	R_i^+	p_i^+	Total Cost (Rs./Yr)
1	29	419	0.12	1380.40
2	10	398	0.05	703.20
3	11	118	0.11	325.60
4	8	173	0.06	375.00
5	15	279	0.07	574.60
6	19	445	0.07	969.00

Grand Total Relevant Cost = 4327.80

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

6.1 Conclusions:

In the present study, some coordinated replenishment policies for multi-item inventory systems have been formulated, solution methodologies suggested and illustration presented through numerical examples. The items share a major fixed replenishment cost or compete for a given total inventory investment. For the deterministic demand case, Silver's popular model has been extended for the case of shortages allowed with backlogging and for the case where group discounts are available for ordering several items simultaneously so as to make the total purchase money value equal to or greater than a given value. For the case of stochastic demand, joint replenishment policies have been formulated for the popular periodic review (R, T) and continuous review (r, Q) policies. The demands in lead time are assumed to be distributed normally. For the (r, Q) policy, Das's approach has been applied to include multi-item interaction yielding a budgetary constraint.

In all above case, as might be expected, computational complexities multiply as the number of items in the system increases. The appropriate heuristic approach is suggested to make the models as far as possible computationally simpler.

6.2 Scope for Future Research:

The present work on multi-item coordinated replenishment though brings some popular single item models to the application in a more realistic situation, is still far from complete in that much more is to be done to evolve an operating policy for a practical situation. Joint replenishment models for multi-echelon inventory system or for multi-stage production system will obviously serve many more real life situations as many organization are of such types. The analysis of dynamic demands case will yield the models to control the systems where there is seasonality in the demands of items. It would be quite realistic and purposeful to analyze the multi-item systems where items have varying characteristics, such as some consumable while others perishable, decaying or repairable.

REFERENCES

1. Baker, K.R., 'On Madigan's Approach to the Deterministic Multi-Product Production and Inventory Problem', Management Science, Vol. 12, No.9, May 1970, pp. 636-638.

2. Bomberger, E., 'A Dynamic Programming Approach to a Lot Size Scheduling Problem', Management Science, Vol. 12, No. 11, July 1966, pp. 778-784.

3. Brown, R.G., 'Decision Rules for Inventory Management, Holt Rinehart and Winston, New York, 1967, Chap. 5.

4. Das, C., 'Some Aids for Lot-Size Inventory Control Under Normal Lead Time Demand', AIIE Transactions, March, 1975.

5. Doll, C.L. and D.C. Whybark, 'An Iterative Procedure for the Single-Machine Multi-Product Lot Scheduling Problem', Management Science, Vol. 20, No. 1, Sept. 1973, pp. 50-55.

6. Goyal, S.K., 'Determination of Economic Packaging Frequency for Items Jointly Replenished', Management Science, Vol. 20, No. 2, October, 1973, pp. 232-235.

7. Goyal, S.K. and Belton, A.S., 'On a Simple Method of Determining Order Quantities in Joint Replenishments Under Deterministic Demand', Management Science, Vol. 25, No. 6, June, 1979, pp. 604.

8. Hadley, G., Whitin, T. M. Prentice Hall, Inc., Englewood Cliffs, N.J., 1963, Chap. 4 and 5.

9. Herron, D.P., 'Inventory Management for Minimum Cost', Management Science, 14, 4,B219-235 (1967).

10. Magigan, J.G., 'Scheduling a Multi-Product Single Machine System for an Infinite Planning Period', Management Science, Vol. 14, No. 11, July 1968, pp. 713-719.

11. Naddor, E., Inventory Systems, John Wiley, New York, 1966, pp. 102-105.

12. Nocturno, D.J., 'Economic Ordering Frequency for Several Items Jointly Replenished', Management Science, Vol. 19, No.9, May 1973, pp. 1093-1096.

13. Parker, L.L., 'Economical Order Quantities and Reorder Points with Uncertain Demand', Naval Res. Log. Quart., 11, 4, 351-358 (1964).

14. Peterson, R. and E.A. Silver, Decision Systems for Inventory Management and Production Planning, John Wiley, 1979.

15. Presutti, V.J., and Trepp, R.C., 'More Ado About Economic Order Quantities (EOQ)', Naval Res. Log. Quart. 17, 243, 251 (1970).

16. Rogers, J., 'A Computational Approach to the Economic-Lot Scheduling Problem', Management Science, Vol.4, No.3, April 1958, pp. 264-291.

17. Silver, E.A., 'A Simple Method of Determining Order Quantities in Joint Replenishments Under Deterministic Demand', Management Science, Vol. 22, No. 12, August 1976, pp. 1351-1361.

18. Sivazlian, B.D., 'Dimensional and Computational Analysis in Stationary (S, s) Inventory Problems with Gamma Distributed Demand', Management Science, 17, 307-311 (1971).

19. Shu, F.T., 'Economic Ordering Frequency for Two Items Jointly Replenished', Management Science, Vol.17, No. 6, February 1971, pp. 406-410.

20. Snyder, A.D., 'Computation of (S,s) Ordering Policy Parameters', Management Science, 21, 2 223-229 (1974).

IMEP-1900-M-RAY-COO